

Extra force and extra mass from non-compact Kaluza–Klein theory in a cosmological model

J.E. Madriz Aguilar^{1,a}, M. Bellini^{2,b}

¹ Instituto de Física y Matemáticas, AP: 2-82, (58040) Universidad Michoacana de San Nicolás de Hidalgo, Morelia, Michoacán, México

² Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad Nacional de Mar del Plata and Consejo Nacional de Ciencia y Tecnología (CONICET), Funes 3350, (7600) Mar del Plata, Argentina

Received: 1 April 2005 / Revised version: 13 May 2005 /

Published online: 28 June 2005 – © Springer-Verlag / Società Italiana di Fisica 2005

Abstract. Using the Hamilton–Jacobi formalism, we study extra force and extra mass in a recently introduced non-compact Kaluza–Klein cosmological model. We examine the inertial 4D mass m_0 of the inflaton field on a 4D FRW bulk in two examples. We find that m_0 has a geometrical origin and antigravitational effects on a non-inertial 4D bulk should be a consequence of the motion of the fifth coordinate with respect to the 4D bulk.

PACS. 04.20.Jb, 11.10.Kk, 98.80.Cq

1 Introduction

The extra dimension is already known to be of potential importance for cosmology [1,2]. During the last years there were many attempts to construct a consistent brane world (BW) cosmology [3]. On the other hand, the induced-matter, or space-time-matter (STM) theory stands out for its closeness to the Einstein’s project of considering matter and radiation as manifestations of pure geometry [4]. Indeed, the gist of the whole theory is to assert that, by embedding the ordinary space-time into a five-dimensional vacuum space, it is possible to describe the macroscopic properties of matter in geometrical terms. Picking up several examples of cosmological and gravitational models, the theory shows how to interpret the energy-momentum tensor corresponding to some standard matter configurations in terms of the geometry of the five-dimensional vacuum space. It was recently questioned in a recent article [5]. The induced-matter theory is sometimes given the name of Kaluza–Klein non-compact field gravity, since Klein’s compactness condition [6] is dropped from the basic assumptions of the theory.

The idea that matter in four dimensions (4D) can be explained from a 5D Ricci-flat ($R_{AB} = 0$) Riemannian manifold is a consequence of Campbell’s theorem. It says that any analytic N -dimensional Riemannian manifold can be locally embedded in a $(N + 1)$ -dimensional Ricci-flat manifold. This is of great importance for establishing the generality of the proposal that 4D field equa-

tions with sources can be locally embedded in 5D field equations without sources [7]. On the other hand, recently it was remarked by Anderson that such a theorem lends only inadequate support, both because it offers no guarantee of continuous dependence on the data and because it disregards causality [5]. In his paper he points out that the theorem is only valid for analytic functions which renders it inappropriate for the study of the relativistic field equations in modern physics. However, analytic functions seem only inappropriate to describe some topological defects like black holes, but not to study cosmological models for which the manifold is global and there are no singularities. Another recent contribution to this issue was made by Katzourabis [8], who demonstrated a global generalization of *Campbell’s theorem, removing the assumption of locality*: Any pseudo-Riemannian analytic N -manifold can be embedded (the whole structure) naturally and isometrically into $(N + 1)$ bulk with arbitrarily prefixed topological structure, fibred over the given one, with vanishing Ricci curvature and torsion of some connection locally compatible with the global metric on the bulk. More generally he found that any 4D manifold with a global tensor field representing the solution of 4D general relativity equations can be embedded naturally and isometrically into a $(4 + d)$ -manifold ($d \in \mathbf{N}$) Ricci-flat and torsion free, where the global topology, the differential structure and the dimensionality of the bulk can be fixed arbitrarily. Briefly stated, 4D phenomenological matter can be induced by a 5D apparent vacuum in the framework of cosmological models where the universe is described by a 3D spatially isotropic, homogeneous and flat (analytic) metric.

^a e-mail: edgar@itzel.ifm.umich.mx

^b e-mail: mbellini@mdp.edu.ar

An important question in STM and BW theories can be split up in two parts.

(1) How to extract the correct 4D interpretation from geometrical objects, like scalar fields which appear in more than four dimensions?

(2) How to predict observable effects from the extra dimensions? There are at least two ways to measure the effect, one which shows no change [9] and one which does [10]. For example, an attempt to understand the first of these questions in the framework of inflationary cosmology [which is governed by the neutral scalar (inflaton) field], from a 5D flat Riemannian manifold was made in [11].

In the last years extra force and extra mass have been subjects of study [12]. It should be an observable effect from extra dimensions on the 4D space-time. The aim of this work is to extend the Hamilton–Jacobi formalism developed by Ponce de Leon [9] to cosmological models where the expansion of the universe is governed by a single inflaton field. This interpretation has the advantage of being free from the complications and ambiguities of the geodesic approach. The formalism provides an unambiguous expression for the rest mass and its variation along the motion as observed in 4D. It is independent of the coordinates and any parametrization used along the motion.

2 Cosmological approach

In a cosmological context, the energy of scalar fields has been argued to contribute to the expansion of the universe [13], and has been proposed to explain inflation [14] as well as the presently observed accelerated expansion [15]. In this paper we shall consider a cosmological model governed by a neutral scalar field that initially suffers an inflationary expansion that has a change of phase towards a decelerated (radiation and later matter dominated) expansion that thereafter evolves towards the observed present day (quintessential) expansion.

We consider the recently introduced [16] 5D metric

$$dS^2 = \epsilon (\psi^2 dN^2 - \psi^2 e^{2N} dr^2 - d\psi^2), \quad (1)$$

where $dr^2 = dx^2 + dy^2 + dz^2$. In order to describe cosmological models we shall consider a 3D spatially isotropic, homogeneous and flat metric (i.e., $x^2 = y^2 = z^2$). Here, the coordinates (N, \mathbf{r}) are dimensionless, the fifth coordinate ψ has spatial unities and ϵ is a dimensionless parameter that can take the values $\epsilon = 1, -1$. The metric (1) describes a flat 5D manifold in apparent vacuum ($G_{AB} = 0$). With the aim to describe neutral matter in a 5D geometrical vacuum (1), as in [17], we can consider the Lagrangian

$${}^{(5)}L(\varphi, \varphi, A) = -\sqrt{\left| \frac{{}^{(5)}g}{{}^{(5)}g_0} \right|} {}^{(5)}\mathcal{L}(\varphi, \varphi, A), \quad (2)$$

where $|{}^{(5)}g| = \psi^8 e^{6N}$ is the absolute value of the determinant for the 5D metric tensor with components g_{AB} (A, B take the values 0, 1, 2, 3, 4) and $|{}^{(5)}g_0| = \psi_0^8 e^{6N_0}$ is a constant of dimensionalization determined by $|{}^{(5)}g|$ evaluated

at $\psi = \psi_0$ and $N = N_0$. We shall consider $N_0 = 0$, so that ${}^{(5)}g_0 = \psi_0^8$. We define the vacuum as a purely kinetic 5D Lagrangian on a globally 5D flat metric [in our case, the metric (1)] [17]. Since the 5D metric (1) describes a manifold in apparent vacuum, the density Lagrangian \mathcal{L} in (2) must be

$${}^{(5)}\mathcal{L}(\varphi, \varphi, A) = \frac{1}{2} g^{AB} \varphi, A \varphi, B, \quad (3)$$

which represents a free scalar field. In the 3D comoving frame $U^r = 0$, the geodesic dynamics $\frac{dU^C}{dS} = -\Gamma_{AB}^C U^A U^B$ with $g_{AB} U^A U^B = 1$ gives us the velocities

$$\begin{aligned} U^\psi &= -\frac{1}{\sqrt{u^2(N) - 1}}, \\ U^r &= 0, \\ U^N &= \frac{u(N)}{\psi \sqrt{u^2(N) - 1}}, \end{aligned} \quad (4)$$

which are satisfied for $S(N) = \pm|N|$. We shall consider the case $S(N) = |N|$. In this representation $\frac{d\psi}{dN} = \psi/u(N)$. Thus the fifth coordinate evolves as

$$\psi(N) = \psi_0 e^{\int dN/u(N)}. \quad (5)$$

Here, ψ_0 is a constant of integration that has spatial unities. From the mathematical point of view, we are taking a foliation of the 5D metric (1) with r constant.

In order to obtain equations of practical use, we can introduce the action $\mathcal{S}(x^A)$ as a function of the generalized coordinates x^A . Hence, since the momentum $P^A = -\frac{\partial \mathcal{S}}{\partial x^A}$, for a diagonal tensor metric g^{AB} , we obtain the Hamilton–Jacobi equation

$$g^{AB} \left(\frac{\partial \mathcal{S}}{\partial x^A} \right) \left(\frac{\partial \mathcal{S}}{\partial x^B} \right) = M_{(5)}^2, \quad (6)$$

where $M_{(5)}$ is the invariant 5D gravitational mass of the object under study (in our case, the mass of the inflaton field). In the particular frame (4), with the Lagrangian (2) and (3), $M_{(5)}$ describes the 5D mass of the scalar field φ in the frame (4). In this case the tensor metric is symmetric (and diagonal), and the Hamilton–Jacobi equation (6) adopts the particular form

$$g^{NN} \left(\frac{\partial \mathcal{S}}{\partial \varphi, N} \right)^2 + g^{\psi\psi} \left(\frac{\partial \mathcal{S}}{\partial \varphi, \psi} \right)^2 = M_{(5)}^2. \quad (7)$$

3 Extra force and extra mass

In general, the line element (1) can be written as

$$dS^2 = ds^2 + dS_{(4)}^2, \quad (8)$$

where ds^2 describes the 4D line element and $dS_{(4)}^2$ only the line element related with the fifth coordinate. We shall define the extra force

$$F^{\text{ext}} = \frac{dP^{x^4}}{ds}, \quad (9)$$

as the force on the submanifold ds^2 due to the motion of the fifth coordinate. In general, P^{x^4} is defined as

$$P^{x^4} = \frac{\partial^{(5)}L}{\partial\varphi_{,x^4}}. \quad (10)$$

In the frame (4) $P^{x^4} \equiv P^\psi$, and it is given by

$$P^\psi = -\frac{\psi^4 e^{3N}}{\psi_0^2} (g^{\psi\psi})^2 \varphi_{,\psi},$$

which also can be written in terms of the potential:

$$P^\psi = -\frac{\psi^4 e^{3N}}{\psi_0^2} g^{\psi\psi} \frac{\partial V(\varphi)}{\partial\varphi_{,\psi}}, \quad (11)$$

where $V(\varphi)$ depends on the particular frame of the observer. In the next subsections we shall see different forms of it in two different frames. Hence, in the frame (4) the extra force

$$F^{\text{ext}} = \frac{\psi^3 e^{3N}}{\psi_0^2} \left(3 \frac{\overset{*}{\psi}}{\psi} \varphi_{,\psi} + 3\varphi_{,\psi} + \overset{*}{\varphi}_{,\psi} \right) \quad (12)$$

holds, where the overstar denotes the derivative with respect to N .

On the other hand, from the equation $g_{AB}U^A U^B = 1$, we obtain the invariant 5D mass $M_{(5)}$:

$$g_{AB}P^A P^B = M_{(5)}^2, \quad (13)$$

where $P^A = M_{(5)}U^A$ is the momentum. For example, in the frame (4) the 4D mass m_0 and the 5D invariant mass $M_{(5)}$ are given respectively by

$$M_{(5)}^2 = g^{NN} \left(\frac{\partial\mathcal{S}}{\partial\varphi_{,N}} \right)^2 + g^{\psi\psi} \left(\frac{\partial\mathcal{S}}{\partial\varphi_{,\psi}} \right)^2, \\ m_0^2 = g^{NN} \left(\frac{\partial\mathcal{S}}{\partial\varphi_{,N}} \right)^2, \quad (14)$$

so that the difference

$$m_0^2 - M_{(5)}^2 = -g^{\psi\psi} \left(\frac{\partial\mathcal{S}}{\partial\varphi_{,\psi}} \right)^2 \quad (15)$$

is non-zero. The interesting thing here is that $m_0^2 > M_{(5)}^2$. In other words, in the frame (4) the motion of the fifth coordinate has an antigravitational effect on the field φ on the submanifold (or bulk) ds^2 . In the next sections we shall study some examples which could be relevant in cosmological models.

3.1 A 4D FRW cosmology

We consider the transformations $t = \int \psi(N)dN$, $R = r\psi$, $L = \psi(N) e^{-\int dN/u(N)}$, such that, for $\psi(t) = 1/h(t)$, we obtain the 5D metric

$$dS^2 = \epsilon \left(dt^2 - e^{2\int h(t)dt} dR^2 - dL^2 \right),$$

where $L = \psi_0$ is a constant and

$$h^2(t) = \left(\dot{b}/b \right)^2 = (8/3)\pi G \langle \rho \rangle$$

is the effective Hubble parameter defined by means of the effective scale factor of the universe b . In this frame (R, t, L) , the velocities $\hat{U}^A = \frac{\partial x^A}{\partial x^B} U^B$ are

$$U^t = \frac{2u(t)}{\sqrt{u^2(t) - 1}}, \quad U^R = -\frac{2r}{\sqrt{u^2(t) - 1}}, \quad U^L = 0, \quad (16)$$

where the old velocities U^B are U^N , $U^r = 0$ and U^ψ and the velocities \hat{U}^B are constrained by the condition

$$\hat{g}_{AB}\hat{U}^A\hat{U}^B = 1. \quad (17)$$

The variables (t, R, L) have physical meaning, because t is the cosmic time and (R, L) are spatial variables. Since the line element is a function of time t (i.e., $S \equiv S(t)$), the new coordinate R gives us the physical distance between galaxies separated by cosmological distances: $R(t) = r/h(t)$. Note that for $r > 1$ ($r < 1$), the 3D spatial distance $R(t)$ is defined on super (sub) Hubble scales. Furthermore $b(t)$ is the effective scale factor of the universe and describes its effective 3D Euclidean (spatial) volume. Hence, the effective 4D metric is a spatially (3D) flat FRW one,

$$dS^2 \rightarrow ds^2 = \epsilon \left(dt^2 - e^{2\int h(t)dt} dR^2 \right), \quad (18)$$

and has an effective 4D scalar curvature ${}^{(4)}\mathcal{R} = 6(\dot{h} + 2h^2)$. The metric (18) has a metric tensor with components $g_{\mu\nu}$ (μ, ν take the values 0, 1, 2, 3). The absolute value of the determinant for this tensor is $|{}^{(4)}g| = (b/b_0)^6$. The density Lagrangian in this new frame was obtained in a previous work [17]:

$${}^{(4)}\mathcal{L}[\varphi(\mathbf{R}, t), \varphi_{,\mu}(\mathbf{R}, t)] \\ = \frac{1}{2} g^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} - \frac{1}{2} \left[(Rh)^2 - \frac{b_0^2}{b^2} \right] (\nabla_R \varphi)^2, \quad (19)$$

and the equation of motion for φ yields

$$\ddot{\varphi} + 3h\dot{\varphi} - \frac{b_0^2}{b^2} \nabla_R^2 \varphi \\ + \left[\left(4\frac{h^3}{h} - 3\frac{\dot{h}}{h} - 3\frac{h^5}{h^2} \right) \dot{\varphi} + \left(\frac{b_0^2}{b^2} - h^2 R^2 \right) \nabla_R^2 \varphi \right] = 0. \quad (20)$$

From (19) and (20), we obtain respectively the effective scalar 4D potential $V(\varphi)$ and its derivative with respect to $\varphi(\mathbf{R}, t)$:

$$V(\varphi) \equiv \frac{1}{2} \left[(Rh)^2 - \left(\frac{b_0}{b} \right)^2 \right] (\nabla_R \varphi)^2, \quad (21)$$

$$V'(\varphi) \equiv \left(4\frac{h^3}{h} - 3\frac{\dot{h}}{h} - 3\frac{h^5}{h^2} \right) \dot{\varphi} \\ + \left(\frac{b_0^2}{b^2} - h^2 R^2 \right) \nabla_R^2 \varphi, \quad (22)$$

where the prime denotes the derivative with respect to φ . Equations (19) and (20) describe the dynamics of the inflaton field $\varphi(\mathbf{R}, t)$ in the metric (18) with the Lagrangian

$${}^{(4)}\mathcal{L}[\varphi(\mathbf{R}, t), \varphi_{,A}(\mathbf{R}, t)] = -\sqrt{\left|\frac{{}^{(4)}g}{{}^{(4)}g_0}\right|} \left[\frac{1}{2} g^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} + V(\varphi) \right], \quad (23)$$

where $|\frac{{}^{(4)}g}{{}^{(4)}g_0}| = 1$.

Furthermore, the function u can be written as a function of time, $u(t) = -\frac{\dot{h}^2}{h}$, where the overdot represents the derivative with respect to the time. The solution $N = \text{arctanh}[1/u(t)]$ corresponds to a time dependent power-law expanding universe, $h(t) = p(t)t^{-1}$, such that the effective scale factor goes as $b \sim e^{\int p(t)/t dt}$. A model for the evolution of the universe was recently developed in [18]. Furthermore, the 4D energy density ρ and the pressure p are [16]

$$8\pi G \rho = 3h^2, \quad (24)$$

$$8\pi G p = -(3h^2 + 2\dot{h}). \quad (25)$$

Note that the function $u(t)$ can be related to the deceleration parameter $q(t) = -\ddot{b}/\dot{b}^2$: $u(t) = 1/[1 + q(t)]$. In what follows we shall consider $\epsilon = 1$ when the universe is accelerated ($q < 0$) and $\epsilon = -1$ in epochs when it is decelerated ($q > 0$). From the condition (17) we can differentiate some different stages of the universe. If $u^2(t) = \frac{4r^2(b/b_0)^2 - 1}{3} > 1$, we obtain that r can take the values $r > 1$ ($r < 1$) for $b/b_0 < 1$ ($b/b_0 > 1$), respectively. Note that now the condition (17) implies that $r \equiv r(t)$, for a given $h(t)$. In this case $q < 0$, so that the expansion is accelerated. On the other hand if $u^2(t) = \frac{4r^2(b/b_0)^2 - 1}{3} < 1$, r can take the values $r < 1$ ($r > 1$) for $b/b_0 > 1$ ($b/b_0 < 1$), respectively. In this stage $q > 0$ and the expansion of the universe is decelerated, so that the function $u(t)$ takes the values $0 < u(t) < 1$ and the velocities (16) become imaginary. Thus, the metric (18) shifts its signature from $(+, -, -, -)$ to $(-, +, +, +)$. When $u(t) = 1$ the deceleration parameter becomes zero because $\dot{b} = 0$. At this moment the velocities (16) rotate synchronically in the complex plane and r takes the values $r = 1$ or $r < 1$ for $b/b_0 = 1$ or $b/b_0 > 1$, respectively. On the other hand, the effective 4D energy density operator ρ is

$$\rho = \frac{1}{2} \left[\dot{\varphi}^2 + \frac{b_0^2}{b^2} (\nabla\varphi)^2 + 2V(\varphi) \right]. \quad (26)$$

Hence, the 4D expectation value of the Einstein equation

$$\left(\frac{\dot{b}}{b}\right)^2 = \frac{8\pi G}{3} \rho$$

on the 4D FRW metric (18) will be

$$\langle h^2 \rangle = \frac{4\pi G}{3} \left\langle \dot{\varphi}^2 + \frac{b_0^2}{b^2} (\nabla\varphi)^2 + 2V(\varphi) \right\rangle, \quad (27)$$

where G is the gravitational constant and $\langle H^2 \rangle \equiv h^2 = \dot{b}^2/b^2$.

In this frame the 5D momentum P^L is null: $P^L = 0$. This implies that the extra force will be

$$F_{\text{ext}} = 0. \quad (28)$$

It also can be viewed from the point of view of the extra mass. In this frame $m_0^2 = M_{(5)}^2$ where

$$\left(\frac{\partial\mathcal{S}}{\partial\varphi_{,t}}\right)^2 - e^{2\int h(t)dt} \left(\frac{\partial\mathcal{S}}{\partial\varphi_{,R}}\right)^2 = M_{(5)}^2. \quad (29)$$

Hence, the inertial 4D mass m_0 is the same as the invariant 5D mass $M_{(5)}$, so that there is no extra force on the effective 4D frame. This can be justified from the fact that the fifth coordinate L do not varies in this frame. In other words the 4D bulk ds^2 is the same as the 5D manifold dS^2 , because $dS_{(4)} = 0$ for an observer that ‘‘expands with the universe’’ in an inertial frame.

3.2 A frame with variable fifth coordinate

Another interesting frame can be described by means of the transformation $t = \int \psi(N) dN$, $R = r\psi$ and $\xi = \psi(N)e^{\int H(N)/H(N)dN}$, so that the 5D velocities are

$$U^t = \frac{2u(t)}{\sqrt{u^2(t) - 1}}, \quad (30)$$

$$U^R = -\frac{2r}{\sqrt{u^2(t) - 1}}, \quad (31)$$

$$U^\xi = \frac{u(t)}{\sqrt{u^2(t) - 1}} \left(\frac{\dot{H}}{hH} - \frac{\dot{h}}{h^2} \right) \frac{H}{H_0}. \quad (32)$$

In this frame the 5D line element is given by

$$dS^2 = \epsilon \left(dt^2 - e^{2\int h(t)dt} dR^2 - \left(\frac{H_0}{H}\right)^2 d\xi^2 \right), \quad (33)$$

where the 4D line element (or ‘‘bulk’’) ds^2 is given by the first two terms in (33):

$$ds^2 = \epsilon \left(dt^2 - e^{2\int h(t)dt} dR^2 \right), \quad (34)$$

and $h^2(t) = H^2(t) + \frac{C}{3}$ for a given constant C . Hence, the extra force on the 4D bulk will be $F^{\text{ext}} = \frac{dP^\xi}{ds}$. Note that the extra force comes from the motion of the fifth coordinate in the effective 4D bulk. In other words, an observer in the 4D bulk (34) will move under the influence of an extra force that, in the example here studied, takes the form

$$F^{\text{ext}} = \left(\left| 1 - \frac{r^2 \dot{h}^2}{h^4} e^{2\int h dt} \right| \right)^{-1/2} \frac{dP^\xi}{dt}, \quad (35)$$

which is invariant under changes of signature (i.e., $\epsilon = 1 \rightarrow \epsilon = -1$). The 5D Lagrangian in this frame takes the form

$${}^{(5)}L(\varphi, \varphi_{,A}) = - \left(\frac{b}{b_0}\right)^3 \frac{H_0}{H} \left(\frac{1}{2}g^{\alpha\beta}\varphi_{,\alpha}\varphi_{,\beta} + V(\varphi)\right), \quad (36)$$

so that the momentum P^ξ is

$$P^\xi = - \left(\frac{b}{b_0}\right)^3 \frac{H_0}{H} g^{\xi\xi} \frac{\partial V(\varphi)}{\partial \varphi_{,\xi}}. \quad (37)$$

In this representation the potential $V(\varphi)$ assumes the form

$$V(\varphi) = \frac{1}{2} \left[(Rh)^2 - \left(\frac{b_0}{b}\right)^2 \right] (\nabla_R \varphi)^2 - \frac{1}{2} \left(\frac{H}{H_0}\right)^2 \varphi_{,\xi}^2 - \frac{H}{H_0} (Rh) \varphi_{,\xi} \nabla_R \varphi, \quad (38)$$

so that the momentum P^ξ is

$$P^\xi = \left(\frac{b}{b_0}\right)^3 \left[\left(\frac{H}{H_0}\right) \varphi_{,\xi} + (Rh) \nabla_R \varphi \right]. \quad (39)$$

Note that the effective kinetic component in the 5D Lagrangian (36) is 4D, but the potential (38) is evaluated in the 5D frame (30), (31) and (32). From (35) and (39), we obtain the extra force for this frame

$$F^{\text{ext}} = \left(\frac{b}{b_0}\right)^3 \left[\left[1 - \left(\frac{R\dot{h}}{h}\right)^2 \left(\frac{b}{b_0}\right)^2 \right]^{-1/2} \times \left[\left(3\frac{\dot{b}}{b} \frac{H}{H_0} + \frac{\dot{H}}{H} \right) \varphi_{,\xi} + \left(3\frac{\dot{b}}{b} (Rh) + (\dot{R}h + R\dot{h}) \right) \nabla_R \varphi + \frac{H}{H_0} \frac{d}{dt} (\varphi_{,\xi}) + (Rh) \frac{d}{dt} (\nabla_r \varphi) \right] \right], \quad (40)$$

where $\left(\frac{b}{b_0}\right)^2 = e^{2\int h dt}$. The important fact here is that the extra force is originated in the last two terms of the 5D potential (38), which depends on the fifth coordinate ξ .

On the other hand the 4D squared mass of the inflation field φ on the 4D bulk (34) is given by

$$m_0^2 = \left(\frac{\partial \mathcal{S}}{\partial \varphi_{,t}}\right)^2 - e^{-2\int h dt} \left(\frac{\partial \mathcal{S}}{\partial \varphi_{,R}}\right)^2, \quad (41)$$

so that one obtains

$$m_0^2 - M_{(5)}^2 = \left(\frac{H}{H_0}\right)^2 \left(\frac{\partial \mathcal{S}}{\partial \varphi_{,\xi}}\right)^2, \quad (42)$$

which gives $m_0^2 \geq M_{(5)}^2$ because the right hand of (42) is positive (for $C > 0$). This is an important result which

shows that the motion of the fifth coordinate has an anti-gravitational effect on an observer in a 4D bulk in which the inflaton field has a 4D mass m_0 . This fact should be responsible for the extra force (40) because the observer “is placed” in a non-inertial frame (or 4D bulk). In this framework the motion of the fifth coordinate is viewed on the bulk as an extra force. Note that it becomes zero as $C \rightarrow 0$, because in this limit $U^\xi \rightarrow 0$ and

$$V(\varphi) \rightarrow \frac{1}{2} \left[(Rh)^2 - \left(\frac{b_0}{b}\right)^2 \right] (\nabla_R \varphi)^2.$$

On the other hand, $U^\xi \rightarrow 0$ as $t \rightarrow \infty$, because $\dot{H} < 0$ (and $\dot{h} < 0$) along all the history of the universe, such that $\left(\frac{H}{H_0}\right)_{t \rightarrow \infty} \rightarrow 0$. Hence, for very late times the external force (40) on the bulk becomes negligible. However, this force should be very important in the early universe when $H/H_0 \gg 1$ and the equation of state is $p \simeq -\rho$, p and ρ being respectively the pressure and energy density. Note that H_0 is the value of the Hubble parameter at the end of inflation.

To illustrate the formalism we can consider the case where $h(t) = t^{-1} p_1(t)$ and $H(t) = t^{-1} p(t)$, where

$$p_1(t) = \sqrt{(2/3 + At^{-2} - Bt^{-1})^2 + \frac{C}{3}t^2}, \quad (43)$$

$$p(t) = \sqrt{2/3 + At^{-2} - Bt^{-1}}. \quad (44)$$

Here $A = 1.5 \cdot 10^{30} G^1$, $B = 10^{15} G^{1/2}$ and we take the special case where the constant C is the cosmological constant Λ : $\Lambda = 1.5 \cdot 10^{-121} G^{-1}$. Furthermore, $G = M_P^{-2}$ is the gravitational constant and $M_P = 1.2 \times 10^{19} \text{ GeV}$ is the Planckian mass [18]. The function $p_1(t)$ represents the power of expansion of the universe in Planckian times. For early times $p_1(t) \gg 1$ and the universe is accelerated ($\ddot{b} > 0$). At $t_r \simeq 10^{16} G^{1/2}$, after inflation ends, the universe becomes radiation dominated: $p_1(t_r) \simeq 1/2$ and after it (for $t \gg t_r$) matter dominated: $p_1(t \gg t_r) \simeq 2/3$. However, at $t \simeq 10^{60.22} G^{1/2}$ the cosmological constant begins to become determinant and the universe becomes almost vacuum dominated (quintessential expansion): $p \lesssim -\frac{2}{3}\rho$. The function $p(t)$ represents the power of expansion of the universe without the cosmological constant. The difference between p_1 and p becomes notorious for very large time (i.e., for $t > 10^{60.22} G^{1/2}$). Numerical calculations give us the time for which $\dot{b} = q = 0$ at the end of inflation: $x(t_0) \simeq 14.778$ [we take $x(t) = \log_{10}(t)$]. At this moment $N(t_0) = 0$, but afterwards it becomes positive. Furthermore, for $x(t) > x(t_*)$ [with $x(t_*) \simeq 60.22$], p_1 begins to increase from the value $p_1 \simeq 2/3$ and the 4D bulk universe is accelerated. In other words, as we demonstrated in a previous work [18], the present day observed (quintessential) acceleration of the universe is a consequence of the non-zero Λ , and has been notorious since the universe was nearly 4 billion years old (when the supernovae explosions occurred).

4 Final comments

In this work we have studied the possible origin of extra force and extra mass from a non-compact Kaluza–Klein formalism recently introduced by using the Hamilton–Jacobi formalism in the framework of cosmological models. However, it is one of the two possible outcomes of a study of this topic (see, for example, [10]). We have examined the inertial 4D mass m_0 of the inflaton field on a 4D FRW bulk in two examples. In the first one $ds^2 = dS^2$, so that the inertial mass m_0 is the same as the 5D gravitational mass $M_{(5)}$ of the inflaton field. As a consequence of this fact there are no extra forces on the 4D bulk. However, in the second example antigravitational effects on a non-inertial 4D bulk should be a consequence of the motion of the fifth coordinate with respect to this bulk, because $dS^2 \neq ds^2$ so that $m_0^2 > M_{(5)}^2$. This disagreement between the 4D inertial and 5D gravitational masses is viewed on the 4D bulk as an extra force. The important point here is that m_0 has a geometrical origin and depends on the frame of the observer. However, $M_{(5)}$ is a 5D invariant gravitational mass and does not depend on the frame of the observer. In other words, all test particles travel on five-dimensional geodesics but observers, who are bounded to space-time, have access only to the 4D part of the trajectory. Finally, in the cosmological model here studied, we find that both, the discrepancy between m_0 and $M_{(5)}$ and extra force (on cosmological scales), are bigger in the early universe [i.e., during inflation ($x(t) < 14.778$)], but become negligible for large times. However, from the point of view of experimentation, at present such a discrepancy should be more notorious on astrophysical scales, where gravitational instabilities are important.

Acknowledgements. The authors acknowledge Dr. J. Ponce de Leon for useful ideas. JEMA acknowledges CONACyT and IFM of UMSNH (México) for financial support. MB acknowledges CONICET, AGENCIA and UNMdP (Argentina) for financial support.

References

1. J. Ponce de León, *Gen. Relativ. Gravit.* **20**, 539 (1998)
2. J.M. Overduin, P.S. Wesson, *Phys. Rep.* **283**, 303 (1997); M.J. Duff, *Int. J. Mod. Phys. A* **11**, 5623 (1996)
3. N. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Lett. B* **429**, 263 (1998); I. Antoniadis, N.A. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Lett. B* **436**, 257 (1998); L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999); **83**, 4690 (1999); F. Dahia, C. Romero, *Class. Quant. Grav.* **21**, 927 (2004)
4. A. Einstein, *The meaning of relativity* (Princeton University Press, Princeton 1956); J. Weeler, *Einstein's vision* (Springer, Berlin 1968); A. Salam, *Rev. Mod. Phys.* **52**, 525 (1968)
5. E. Anderson, The Campbell-Magaard theorem is inadequate and inappropriate as a protective theorem for relativistic equation, gr-qc/0409122
6. T. Kaluza, *Sitz. Preuss. Akad. Wiss.* **33**, 966 (1921); O. Klein, *Z. Phys.* **37**, 895 (1926)
7. P.S. Wesson, *Space-Time-Matter* (World Scientific, Singapore 1999)
8. N.I. Katzourakis, Bundly-Theoretical globalization of Campbell-Magaard embedding theorem in the context of MD gravity, math-ph/0407067
9. J. Ponce de Leon, *Int. J. Mod. Phys. D* **12**, 757 (2003); *Gen. Rel. Grav.* **35**, 1365 (2003); **36**, 1333 (2004); see also J. Ponce de Leon, *Grav. Cosmol.* **8**, 272 (2002)
10. B. Mashhoon, P.S. Wesson, H. Liu, *Gen. Rel. Grav.* **30**, 555 (1998); P.S. Wesson, B. Mashhoon, H. Liu, W.N. Sajko, *Phys. Lett.* **456**, 34 (1999)
11. Mauricio Bellini, *Nucl. Phys. B* **660**, 389 (2003)
12. S.S. Seahra, P.S. Wesson, *Gen. Rel. Grav.* **33**, 1731 (2001); F. Dahia, E.M. Monte, C. Romero, *Mod. Phys. Lett. A* **18**, 1773 (2003)
13. J. Dreitlein, *Phys. Rev. Lett.* **33**, 1243 (1974); H.W. van Holten, *Phys. Rev. Lett.* **89**, 202301 (2002)
14. A. Guth, *Phys. Rev. D* **23**, 347 (1981)
15. R.R. Caldwell, R. Dave, P.J. Steinhardt, *Phys. Rev. Lett.* **80**, 1582 (1998); C. Armendariz-Picon, V. Mukhanov, P.J. Steinhardt, *Phys. Rev. Lett.* **85**, 4438 (2000)
16. D.S. Ledesma, M. Bellini, *Phys. Lett. B* **581**, 1 (2004)
17. J.E. Madriz Aguilar, M. Bellini, *Eur. Phys. J. C* **38**, 123 (2004); *Phys. Lett. B* **596**, 116 (2004)
18. J.E. Madriz Aguilar, M. Bellini, *Eur. Phys. J. C* **38**, 367 (2004)